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An Analysis of the Sensitivity of Fish Biomass Estimates to the Equivalent Beam Angle Estimate

R. Shotton

Marine Ecology Laboratory
Bedford Institute of Oceanography
Dartmouth, Nova Scotia

ABSTRACT

A comparison is made of the results given by different estimators of the Equivalent Beam Angle with those obtained by digitizing transducer directivity patterns and empirically determining the Equivalent Beam Angle. With one exception, the estimators used generally show good agreement with the measured Equivalent Beam Angle. Use of an (ideal) beam width defined by the 3 dB points would cause a bias of up to 74% depending on the particular transducer used.

RESUME

Comparaison des résultats obtenus avec différents estimateurs d'angle équivalent du faisceau avec ceux obtenus par conversion numérique des diagrammes directionnels de transducteurs et par détermination empirique de l'angle du faisceau équivalent. À une exception près, les estimateurs utlisés se sont avérés en accord avec l'angle du faisceau équivalent measuré. L'utilisation d'un faisceau de largeur "idéale" défini par les points de coupure à 3 dB entraîne une déviation pouvant atteindre 74% suivant le transducteur utilisé.

INTRODUCTION

Echo integration is a technique of acoustic fish stock assessment which is appropriate when fish form a scattering layer (Forbes and Nakken, 1972; Burczynski, 1979). An assumption made in this technique is that the fish echo intensity is a linear function of the biomass of the fish insonified in the pulse volume. The echo intensity at the transducer face can be determined by:

$$I = I_0 \frac{\frac{\sum^{n} \sigma_i}{4\pi} \frac{c\tau}{2} \int^{2\pi} b(\theta,\phi)^2 \, d\Omega}{R^2 e^{2\alpha R}} \qquad (1)$$

where;

I_0 = source level intensity

σ_i = scattering cross-section of ith fish in the pulse volume

$\frac{c\tau}{2}$ = pulse width

α = attenuation coefficient of sound in seawater

R = range of pulse volume with respect to the transducer

n = number of fish in the pulse volume

$b(\theta,\phi)$ = directivity of the transducer as defined by Urick (1978).

$d\Omega$ = infinetisimal solid angle such that,

$$\int_0^{2\pi} d\Omega = 2\pi$$

The integral of $b(\theta,\phi)$, the two-way transducer directivity pattern, or "system directional factor" of Craig (1979) accounts for the fact that transducers, because of wave interference effects, do not transmit sound co-ordinates is a function of the azimuthal and declination angles. Maximum transmitted sound intensity is on the acoustic axis, i.e., $\theta = \phi = 0°$

The term $\int_0^{2\pi} b(\theta,\phi)^2 \, d\Omega$ may be equivalently expressed as

$$\int_0^{2\pi} \int_0^{\pi/2} b(\theta,\phi)^2 \sin\theta \, d\theta \, d\phi$$

The integral function is referred to variously as:

(1) integrated beam width factor (Clay and Medwin, 1977)

(2) integrated transducer directivity function (Hamilton et al., 1977)

(3) equivalent ideal beam pattern (Bodhott, 1977)

(4) equivalent beam width or equivalent ideal beam width (Urick, 1978)

(5) solid angle of an equivalent ideal transducer (Burczynski, 1979)

(6) effective beam angle for integration (Craig, 1979)

The term is often used undefined (e.g. Forbes and Nakken, 1972; Cushing, 1978). I propose the following usage;

$$\text{Equivalent Beam Angle*} = \int_0^{2\pi} b(\theta,\phi)^2 \, d\Omega$$

The equivalent beam angle has dimensions of steradians.

From equation (1) it is apparent that any bias in the calculation of the equivalent beam angle will cause a similar relative bias in estimates of fish biomass. The purpose of this note is to examine the possible

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* The Oxford English Dictionary defines equivalent as, "Equal in value; having the same effect; corresponding". Effective is defined as, "Having an effect, actual, existing". Hence either word would be applicable, but equivalent seems the more appropriate of the two. The term 'angle' is preferable to 'width' as the unit is of solid angle.

bias in estimates of fish biomass that can occur because of this.

The directivity of the transmitted sound intensity with respect to azimuth and declination is a function of the size and shape of the face of the transducer and the wave-length of the transmitted sound. Since transducers are also used as the receivers in acoustic fish stock assessment, and most transducers have identical transmit and receive beam patterns, then the directivity at any particular angular co-ordinates for both transmission and reception is given by the square of the one-way directivity value.

Because of the relatively minor use of asymmetrical, (i.e., rectangular) transducers in acoustic fish stock assessment, this discussion will be restricted to symmetrical (i.e., circular) transducers. For circular transducers the directivity is uniform in azimuth and is only a function of subtended angle to the acoustic axis. Hence,

$$\text{Equivalent Beam Angle} = 2\pi \int_0^{\pi/2} b(\theta)^2 \sin\theta \, d\theta \qquad (2)$$

Urick (1975) gives the directivity pattern function for a piston transducer in an infinite baffle as

$$b(\theta) = \left[\frac{2J_1\{(\pi D/\lambda)\sin\theta)\}}{(\pi D/\lambda)\sin\theta)}\right]^2$$

where θ is angle with respect to the acoustic axis, $0 \leq \theta \leq \pi/2$; λ is wave length of the transmitted sound; the operator J_1 is a Bessel function of the first kind with order one; and D is the transducer diameter. Many contemporary transducers are constructed by setting a number of small elements in some binding medium so that the sum of their effects approximates that of a piston transducer. The positions of the elements in the transducer face are determined at random, subject to packing constraints. The Ametek 50 kHz transducers examined here have the outer 50% of the element locations randomized in the plane to obtain random phase distribution and enable shading of the side lobes. This reduces the reception of noise from sources such as the ships propeller. For this reason their effective beam angle may differ from that obtained using expression (2).

Although I agree with Craig (1979), that generally there is no justification for logarithmic transformations in acoustics, directively terms are often expressed in decibel or logarithmic form in the "acoustics literature". When the equivalent beam angle is expressed in logarithmic units, I propose the term,

$$\text{Beam Factor (BF)} = 10 \log 2\pi \int_0^{\pi/2} b(\Theta)^2 \sin\Theta \, d\Theta$$

This term is reference 1 steradian.

METHODS

When calculating the Equivalent Beam Angle the transducer can be assumed to act as an ideal piston of diameter equal to that of the transducer. Several approximations to expression (10) are given in the literature. Hamilton, Lozow, Suomala and Werner (1977) give

$$E[b(\theta)^2] = \frac{2}{\pi} \left(\frac{\lambda}{D}\right)^2 \left[1-e^{-(\pi D/\lambda)^2 \theta^2}\right] \qquad (3)$$

Urick (1975; p. 217) gives two approximations for the beam factor for a circular plane array which he terms the logarithmic equivalent two-way beam width;

$$BF = 20 \log \left(\frac{\lambda}{2\pi a}\right) + 7.7 \qquad (4)$$

and

$$BF = 20 \log \theta - 31.6 \qquad (5)$$

A further approximation that can be used is to consider that the beam has unit directivity between the angles at which the intensity is half that (the 3dB points) of the acoustic axis. The solid angle of the beam if its half angle is θ, is 2π (1−cosθ). Then for a beam of unit directivity over this solid angle and zero elsewhere:

Equivalent Beam angle = 2π(1−cosθ), or in logarithmic units,

$$BF = 10 \log_{10} \{2\pi(1-\cos\theta)\}. \qquad (6)$$

The directivity patterns for two transducers of 50 kHz and 120 kHz were digitized and the effective beam angle determined by an approximation method. The 50 kHz transducer consisted of 120 resonators set within a disc 0.309 m in diameter for a beam intended to have a 3° half angle (acoustic axis to the 3dB point). A beam with a 6° half angle can be formed by using the central 32 resonators, which are located so as to form a disc 0.159 m in diameter. The 120 kHz transducer was of the type used with Simrad. This transducer has a diameter of 0.076 m. It should be noted that Anon (1973, p. 2.10) refers to this transducer as having a diameter of 10 cm.

If the digitized co-ordinates tracing the transducer directivity pattern, expressed in decibel units are obtained, then

$$b(\theta)_i = \frac{10^{x_i/10 \sin\theta_i}}{10^{y_i/10}}$$

The denominator is the measure of intensity on the acoustic axis; the numerator the measure of intensity at angle θ_i, where

$$\theta_i = \arctan \frac{x_i}{y_i}$$

Equivalent beam angle $= \Sigma\ b(\theta_i)^2\ f(\theta_i)$

$$0 \leq \theta \leq \frac{\pi}{2}$$

where $f(\theta_i)$ = relative contribution to the total beam at θ_i.

$$= \cos\theta' - \cos\theta''$$

where $\theta' = \dfrac{\theta_i + \theta_{i-1}}{2}$

$$\theta'' = \frac{\theta_i + \theta_{i+1}}{2}$$

With such a method it is not essential that the digitized points be uniformly separated to avoid bias.

DISCUSSION

The results of the different estimates are given in Table 1. Because the Ametek transducer consists of elements set in a disc, the difference between the Equivalent Beam Angle estimate obtained by digitizing the transducer directivity pattern and the other methods could result from the transducer behaviour deviating from that of an ideal "piston" type transducer. With the exception of the estimate based on an ideal beam between the 3 dB points, the differences for the Ametek 3° and 6° half-angle beams was no greater than 0.52 dB and 0.55 dB respectively. This is less than

TABLE 1 - Equivalent Beam Angle Estimates

Transducer	Ametek	BF	Bias%	Ametek	BF	Bias%	Simrad	BF	Bias%
Frequency kHz Diameter m	50 0.305			50 0.159			120 0.076		
Digital Approximation	0.00640	-21.94		0.02000	-16.99		0.01850	-17.33	
Numerical Integration Equation (2)	0.00568	-22.46	-11.3	0.02094	-16.79	4.7	0.01514	-18.20	-18.2
Hamilton <u>et al.</u> 1977 Equation (3)	0.00617	-22.10	-3.6	0.02270	-16.44	13.5	0.01714	-17.66	-7.4
Urick 1978 Equation (4)	0.00578	-22.38	-9.6	0.02128	-16.72	6.4	0.01607	-17.94	-13.1
Urick 1978 Equation (5)	0.00622	-22.06	-2.7	0.02084	-16.81	4.2	0.02328	-16.33	25.8
Ideal Beam (3 dB points)	0.00889	-20.51	39.0	0.02884	-15.40	44.2	0.03214	-14.93	73.7

the error of 1 dB conventionally associated with acoustic system calibration. For the single element transducer used in Simrad systems, excluding the ideal beam estimate, the maximum error was 1.00 dB. Equation (5) of Urick (1978) provided the closest estimate to the digitized estimate for the Ametek transducer. The estimate of Hamilton et al. (1977) was closest for the single element transducer. Anon (1971, p. 2.19) assumes a 10 cm diameter for the Simrad transducer, and a Beam Factor estimate (termed, 10 log ψ) of -18 dB. Relative to the estimate obtained by digitizing the transducer directivity pattern this would cause a -14% bias in biomass estimates.

With the exception of the Ideal Beam estimate, the error incurred in using any of the Equivalent Beam Angle estimators discussed here is not likely to be serious relative to uncertainties in the system calibration or knowledge of the actual scattering cross-section of the insonified fish.

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